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Multidimensional Liquidity and Momentum Augmented Fama and French (2015) Five-factor Model: Evidence From Pakistan

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Multidimensional Liu (2006) liquidity; Momentum; Fama & French (2015) five-factor model; GRS test; Pakistan Stock Exchange

ABSTRACT

Using a large sample size of 521 firms, this study is the first to evaluate and compare the performance of five prominent factor-pricing models in PSX: the CAPM, Liu's two-factor model, Fama-French three-factor model, Carhart four-factor model, Fama-French five-factor model, and both liquidity and momentum augmented Fama-French five-factor model (seven-factor model) using monthly data from January 2002 to December 2020. The results revealed that the value factor is not redundant in the market. The factor-spanning test confirmed that liquidity and momentum are essential augmentations to FF-5FM by producing statistically significant intercepts in both factor regressions. Similarly, the liquidity and momentum augmented FF-5FM outperforms, even though the absolute average alpha of the GRS test implies that the seven-factor model offered a better explanation. This confirms the spanning tests by demonstrating that when combined with FF-5FM; both factors enhanced the specification's prediction power. Liu's (2006) two-factor and Liquidity enhanced FF-5FM are justifiably the next two superior explanatory models proposed by the GRS test.

INTRODUCTION

In this modern era, the significance of the stock market cannot be ignored in the study of finance and economics which contributes a major part to the economy of a country. It facilitates the transfer of fund-flow from savers to investors and makes the household and institutional investors' decision-making process more convenient and rationale regarding the investing and financing activities. Similarly, one of the most important characteristics of publicly traded equities is the requirement for transparent reporting, which encourages investors to participate in the stock market, (Lee, Cheng, & Chong, 2016).

The high risks associated with capital market securities make investors hyper-conscious of them, forcing them to use better tools and techniques for evaluating equity securities before constructing efficient portfolios to diversify to beat the market, yielding excess realized gains, and thus maximizing wealth, (Ameer, 2013; Brad M Barber & Odean, 2013; Boucher, Jasinski, Kouontchou, & Tokpavi, 2021). On the contrary, they are behaviourally hesitant to invest in illiquid stocks for a variety of reasons. As a result, they seek an illiquidity risk premium from the holding stocks in their portfolios as compensation for the related possible illiquidity risks. However, various valid hypotheses are supported to determine

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stock returns, (Ameer, 2013). Moreover, when investors and portfolio managers apply theory-based Asset Pricing Models (APMs) to real-world financial applications regularly, they can optimize their wealth. As a result, there has been a profusion of substantial theoretically supported and empirically justified evidence concerning the determinants of stock returns evaluated by various risk factors through simple and multiple APMs, all over the world, over the previous few decades. Similarly, to investigate APMs, researchers looked at theoretical and empirically tested studies that documented various anomalies representing the variability in projected stock returns in settled and emerging stock markets around the world.

One of the major areas of finance literature that receives substantial attention from academics and researchers is liquidity. Over the previous four decades, hence, a variety of contemporary low-frequency indicators are classified as liquidity proxies based on the dimension they capture, including transaction cost, volume, price effect, and multi-dimensional-based measures such as (Acharya & Pedersen, 2005; Amihud, 2002; Amihud & Mendelson, 1986; Liu, 2006; Pástor & Stambaugh, 2003; Roll, 1984)[*]. In the same way, a plethora of studies empirically investigated all the dimensions of liquidity and they used liquidity as independent factors (liquidity risk-premium) in augmenting with various standard asset-pricing models, throughout the globe, such as (Acharya & Pedersen, 2005; Liu, 2006; Pástor & Stambaugh, 2003; Sadka, 2006). Moreover, (Liu, 2006) proposed a multidimensional-based measure, which empirically shown to be the robust performer when compared to standard CAPM and FF-3FM, (Ma, Zhang, & Liu, 2021; Minović & Živković, 2012), which is rarely augmented with FF-5FM in emerging equity markets. Ma et al. (2021) argued that because the price-impact-based component has limited pricing capability, the recent asset-pricing literature has mainly ignored liquidity risk. Therefore, this study chooses (Liu, 2006) multidimensional measure as an additional factor along with (Carhart, 1997) momentum factor to examine whether both factors perform better in explaining average stocks portfolios returns using a comprehensive sample from Pakistan Stock Exchange (PSX).

Liquid stocks, in particular, are described as securities that can trade in big volumes, quick, at a cheap cost, and with little price impact. Liu (2006) measure provided ‘turnover-adjusted number of days with nil-trading volume’ which is one of the most broadly recognized descriptions of liquidity which considered several aspects of liquidity, such as trading quantity, speed, and cost with a special focus on trading speed, (Le & Gregoriou, 2020).

[*] See (Le & Gregoriou, 2020) reviews the literature on liquidity measures.

Liu (2006) Liquidity multidimensional specification:

Liu (2006) liquidity measure equation as follows:

$$LM_{it} = \left[NoZV_{it} + \frac{1/(turnover_{it})}{Deflator} \right] \times \frac{21}{NoTD_t} \quad (1)$$

- $NoZV_{it}$ is the number of days with zero volume of stock *I* in month *t*.
- $Turnover_{it}$ is the sum of the daily turnover of stock *I* in month *t*.
- $NoTD_t$ is the total number of trading days in the market in month *t*, and
- *Deflator* is 480000 as suggested by (Liu, 2006).

The rest of the paper is structured as follows. The second section addresses review of the literature. Section 3 displays data and methodology. Sections 4 addresses empirical results and analysis and 5 conclusions.

LITERATURE REVIEW

Over the last three decades, academic scholars throughout the world have conducted a plethora of empirical studies on various asset-pricing models by augmenting numerous anomalies with the standard Capital Asset Pricing Model (henceforth CAPM). The trio (Lintner, 1965; Mossin, 1966; Sharpe, 1964) pioneered independently the theoretical and empirical work of APMs, which is summed up as CAPM. It elaborates that beta solely captures the expected returns of an asset. Convincingly, it postulates the nexus between expected returns and risk (beta). It is extensively utilized for estimating stock returns and movement behavior of stocks, (Lee et al., 2016), as well as measuring the cost of equity capital, (Frank & Shen, 2016; Johnstone, 2020; Schramn & Wang, 1999). More significantly, the risk-return nexus is the primary focus of many asset-pricing models.

Numerous research papers have experimentally studied and claimed that the CAPM linear connection is too flat, which is related to measurement error in beta. (Douglas, 1969) suggested that this nexus is too flat, which is associated with measurement error in beta (Miller & Scholes, 1972). To reduce measurement errors, (Fama & MacBeth, 1973) linked this nexus with portfolio, which supported the argument that this association is near-linear while the slope is still underestimated. (Fama & French, 1992) also revealed a too flat and statistically insignificant risk-return relationship. Furthermore, empirical evidence suggests that (Fama & French, 1993) the three-factor model (henceforth FF-3FM) outperformed single-factor models utilizing data from established and emerging equity markets.

Based on the beta analysis, there are two forms of CAPM: static OLS (where the beta coefficient is constant over time) and time-variable beta coefficient (where the beta coefficient varies over time). (Kassimatis, 2008) discovered that HML and WML behave as though they are disappearing based on time-varying beta while using Australian equities market data; on the other hand, SMB returns are reduced. Furthermore, the findings indicated that the momentum effect is underperforming in the market. The study further advocated that while the business cycle (macroeconomic condition) differs from country to country, so does the return-generating process. Accordingly, assuming beta to be constant may lead to spurious regression findings.

The stream of empirical study began with the development of CAPM but was criticized by numerous academics and practitioners, drawing their attention to a new area of research in the finance and economics literature. In explaining the variance in equity returns, it posits market beta as a monopolistic risk factor, (Ali & Badhani, 2021; Hamaui & Jaffard, 2021). Alternatively, FF-3FM provided a full paradigm shift by augmenting two additional anomalies into the CAPM: size and B|M ratio, diverting academics and practitioners' attention to the firm-specific characteristics. Following that, they began empirically examining FF-3FM in a range of equity markets and discovered it to be the most appropriate APM for explaining cross-sectional anticipated stock returns in both established and emerging equity markets. It is extensively used as a benchmark asset-pricing model, according to (Fan & Liu, 2005). Berk (1995) has criticized it for having misspecification errors between these two extra price-scaled variables, in addition to its support.

(Carhart, 1997) tried to study a different type of momentum anomaly by highlighting the impact of investor behavior in the cyclic movement of equities from winners to losers and back again. The (Carhart, 1997) four-factor model (henceforth C-4FM) was named after it was discovered as the fourth pattern in the asset-pricing literature. The researchers also looked at the influence of the C-4FM, claiming that it is important in understanding stock returns throughout the world, see (ap Gwilym, Clare, Seaton, & Thomas, 2010).

The literature supported by a variety of studies evidenced that FF-5FM estimates expected stock returns more robustly in developed equity markets across the globe. (Fama & French, 2017) examined the five-factor model using developed equity markets data and observed that the model abundantly captures the patterns in mean returns in a global context. Similarly, Elliot, Docherty, Easton, and Lee (2018) also tested the model in the Australian equity market and (Kubota & Takehara, 2018) in the Japanese equity market. However, these regions have a different impact, (Jacobs, 2016) as the investment factor shows significant for the US whereas insignificant for the Chinese equity market, (Guo, Zhang, Zhang, & Zhang, 2017). Furthermore, profitability and investment demonstrate substantial effects for Europe and the Asia Pacific but are redundant for the Japan equity market, (Ali, Khurram, & Jiang, 2021). In the same way, momentum impact is observed insignificant in Asian equity markets including China and Korea (Chui, Titman, & Wei, 2010; Lin, 2017) excluding Japan whereas, in developed markets, it shows the influential impact, (Asness, Moskowitz, & Pedersen, 2013). The FF-5FM performed well in the local version but inefficient in the international version, (Fama & French, 2017), likewise (Foye, 2018; Zaremba, Czapkiewicz, Szczygielski, & Kaganov, 2019; Zaremba & Maydybura, 2019), examined FF-

5FM internationally but evidenced mix findings for emerging equity markets.

In addition to factors augmentation, (Amihud & Mendelson, 1986) were the pioneer to postulate the illiquidity premium into the APMs literature, which should also be compensated with a higher premium. The difficulty in evaluating illiquid assets, the long time horizon to resale, and, most importantly, the higher trading costs caused illiquid assets to be compensated for holding illiquid securities. (Anwar & Hogholm, 2020) argued that many empirical studies have investigated the nexus between illiquidity and stock returns in established and emerging equity markets throughout the world. The prior literature documented liquidity pattern as factor-loading and structured it as a portfolio of low-liquidity (illiquid) firms minus portfolio of high-liquidity (liquid) firms portfolio and observed as resilient and significant for compensating asset pricing anomalies, see (Acharya & Pedersen, 2005; Liu, 2006; Pástor & Stambaugh, 2003).

An overabundance of research studies augmented various adjusted anomalies to CAPM and have been buttressed to examine the efficient predictability and explanatory power of the model such as (Feng, Giglio, & Xiu, 2020) used 150 risk factors, which (Cochrane, 2011) earlier argued as "factor zoo". Among these studies, (Banz, 1981); (Reinganum, 1981) proposed size-pattern, (Bhandari, 1988) leverage-pattern, (Basu, 1983) earnings to price ratio, (Amihud & Mendelson, 1986) liquidity-pattern, (Fama & French, 1993) introduced both size and value patterns, (Carhart, 1997; Jegadeesh & Titman, 1993) proposed momentum, (Fama & French, 2015) proposed profitability and investment patterns, (Maiti & Balakrishnan, 2018; Roy & Shijin, 2019) human-capital component (labor income-growth rate).

Several APMs that include various characteristics or risk factors are being explored in established and emerging equities markets throughout the world, according to Lee (2011). Liquidity is also used as a feature, see (Amihud, 2002; Amihud & Mendelson, 1986; Brennan & Subrahmanyam, 1996), as well as a risk factor, see (Acharya & Pedersen, 2005; Amihud, 2002; Liu, 2006; Pástor & Stambaugh, 2003; Watanabe & Watanabe, 2008). Liquidity plays a significant function in stock markets because of its complexity. It is an intricate problem which traditional models fail to address thoroughly, (Ang, Hodrick, Xing, & Zhang, 2006).

While evaluating an efficient portfolio in a strong efficient equity market, portfolio managers rationally participate in the construction of continuous rebalancing portfolios and diversification of idiosyncratic risk, but investors mostly expect the stock to be resold quickly without reducing its value. A variety of efficient APMs is proposed and empirically tested for efficient portfolio construction, but to the best of our knowledge, (Liu, 2006) multidimensional liquidity factor in augmentation with profitability, investment, and momentum patterns has not been investigated in a single study for analyzing generally developed and particularly emerging equity markets. However, the liquidity factor has a strong effect on the process by which investors and portfolio managers make decisions (Vidović, Poklepović, & Aljinović, 2014). According to (Amihud & Mendelson, 1986), liquidity is a neglected subject in academic study, although (Aitken & Winn, 1997) reported 68 liquidity measures studied by academics. In their research, (Racicot & Rentz, 2016; Racicot, Rentz, & Kahl, 2017) employed (Pástor & Stambaugh, 2003) liquidity factor as the sixth factor.

Liu (2006) establishes a new multidimensional liquidity premium that is robust to the CAPM and FF-3FM using a novel measure of liquidity and exhibits that liquidity is a major determinant of pricing risk. The relationship of the two-factor (market and liquidity) model with the cross-sectional securities returns is explained well, as compared to the size factor, contrarian strategy, and fundamentals such as cash-flow, earnings, and dividend-to-price ratios. Furthermore, it accounts for the B|M impact that FF-3FM cannot explain.

Minović and Živković (2012) studied the influence of market, size, B|M ratio, and liquidity risk factor on projected securities returns in the Serbian stock exchange. They examined the CAPM, FF-3FM, LCAPM by (Liu, 2006), and liquidity augmented FF-3FM factors using daily data for the years 2005 to 2009. They observed that liquidity and size factors had a substantial influence on stock returns determination in Serbia using a rigorous technique and complicated dataset. Moreover, their findings show that the factor relating to the B|M ratio (value) of a firm does not play a significant influence in asset pricing in Serbia. They discovered that Liu's two-factor LCAPM model outperforms the conventional CAPM and FF-3FM in describing stock returns. Furthermore, Liu's LCAPM is perhaps a useful model for calculating realistic asset returns. The augmentation of Liu’s liquidity with FF-3FM and CAPM might help researchers better comprehend equilibrium in the Serbian stock market.

Zada, Rehman, and Khwaja (2018) empirically investigated the FF-5FM using a sample of 120 companies from the general population with the greatest market capitalization. They used the two-step regression approach of (Fama & MacBeth, 1973). They built 16 portfolios based on 14 years of data from Jul 2000 to June 2013 and estimated excess market returns by subtracting 6 months T-Bill rates. Their findings indicated that small, value, robust, and conservative portfolios outperformed big, growth, weak, and aggressive portfolios respectively. Furthermore, they found HML to be redundant in the PSX. By removing the HML factor from the study, the adjusted R-square value improved.

Cox and Britten (2019) studied the FF-5FM in the Johannesburg Securities Exchange (JSE). They investigated FF-3FM and various factors augmented with FF-5FM comparatively. The findings discovered SMB-HML and SMB-RMW performed better results using time-series stock returns. In terms of describing cross-sectional stock returns, FF-5FM performed superior results in the market. Their findings show a substantial inverse SMB-premium and a negative connection between beta and returns, while no significant HML-premium.

Tashfeen, Ullah, and Naeem (2020) examined the investor behavior in PSX. They tested FF-5FM and observed a significant impact of investment whereas poor results for value and profitability factors using monthly data for a sample of 100 firms from 2011 to 2014.

Ma et al. (2021) examined various liquidity risk augmented APMs using Arbitrage pricing theory framework, squared Sharpe ratio matric, (Pástor & Stambaugh, 2003; Sadka, 2006) (PS-2003) liquidity factor and (Liu, 2006) multidimensional liquidity augmented with FF-3FM and FF-5FM and (Hou, Xue, & Zhang, 2015) four-factor model (H-4FM). They revealed that liquidity adjusted-CAPM (LCAPM) does an excellent job of describing the cross-section of projected stock returns while critically argued the weak performance of PS-2003 and Sadka-2006 liquidity results. The findings of their paper confirm the significance of liquidity factor in APMs and conclusively Liu's LCAPM is recognized as an appropriate model.

Yang and Zhang (2021) investigated stock liquidity and transaction costs nexus using NYSE/AMEX/ARCA/NASDAQ data from 1926 to 2011. They examined FF-3FM and FF-5FM, along with LCAPM and PS-2003 liquidity measures. FF-5FM is used to evaluate risk-adjusted-performance. The study revealed that the transaction cost is not a factor in determining the liquidity premium. To do portfolio analysis, the study utilized both weighting portfolios for robustness: equally weighted and value-weighted portfolios. Moreover, they observed that the impact of transaction costs on potential equity returns is unplanned and unimportant.

(Ali et al., 2021) examined the momentum augmented FF-5FM in PSX using an extensive sample of PSX enlisted firms from 2003 to 2016. They examined four baseline asset-pricing models and observed that FF-5FM outperformed FF-3FM and C-4FM. They further investigated that value and momentum factors are redundant using factor spanning-test. Moreover, the investment results observed weak in the market. The GRS test results of the GRS test face failure for many portfolios.

(Khan, Wahid, Rahim, Ali, & Ahmad, 2021) studied the momentum effect in PSX using 466 enlisted firms data from 2009-2017 by employing 25 momentum strategies through Carhart 4 factor model. Using C-4FM, the findings were observed positive and significant with portfolio returns and similar results for market and value factors. On the other hand, size and momentum patterns show negative and significant nexus with portfolio returns in PSX.

RESEARCH METHODOLOGY

The sample of the study includes financial and non-financial (manufacturing and non-manufacturing) firms enlisted on Pakistan Stock Exchange (PSX) by following (Barber & Lyon, 1997; Mirza & Shahid, 2008). This study uses monthly common stock returns data of 521 out of 631 firms from January 2002 through December 2020, based on the filter suggested by (Ardila-Alvarez, Forro, & Sornette, 2021), we excluded stocks subject to non-availability of data for consecutive two years to control for survival bias. The time series simple and multivariate OLS regressions are employed to investigate the nexus of various factors with stock returns based on size and book-to-market ratio portfolios.

Following (Liu, 2006) methodology for measuring multidimensional liquidity (equation-1), 521 firms' data have been evaluated and then portfolios are constructed based on low-liquidity firms, neutral-liquidity firms, and high-liquidity firms are categorized. To calculate the illiquidity premium, high-liquidity (Liquid) firms' average returns are deducted from low-liquidity (Illiquid) firms such as illiquid firms minus liquid firms (IML). Firms having no trading are excluded from the analysis.

The momentum portfolios are constructed based on previous 1-year historical returns by following (Naughton, Truong, & Veeraraghavan, 2008), the firms having higher returns and low returns are classified based on winners and losers respectively. The average returns of lowers firms are subtracted from the average returns of winners firms such as winner firms' return minus loser firms' return to construct the Winners minus Losers (WML) factor.

EMPIRICAL SPECIFICATION

This study examines the following standard single-multivariate statistical standard baseline models:

1. Capital Asset Pricing Model (CAPM):

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \varepsilon_i \qquad (2)$$

In which, R_i-R_f, is expected excess portfolio returns. R_m is the expected market return. β_m is the sensitivities of the market factor.

2. Liquidity augmented CAPM (L-CAPM) or Liu (2006) two-factor model:

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_l(ImL) + \varepsilon_i \qquad (3)$$

In which, ImL is the Illiquid minus Liquid firm's returns called Liquidity-factor. β_l, is the coefficient of liquidity.

3. Momentum augmented CAPM (M-CAPM):

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_w(WmL) + \varepsilon_i \quad (4)$$

In which, WmL is the Winner minus Loser firm's returns called Momentum-factor. β_w, is the coefficient of momentum.

4. Liquidity & Momentum augmented CAPM (LM-CAPM):

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_l(ImL) + \beta_w(WmL) + \varepsilon_i \qquad (5)$$

5. Fama & French (1993) three-factor model (henceforth FF-3FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \varepsilon_i \qquad (6)$$

In which, SmB is the firms having small market-cap minus firms having big market-cap called Size-factor. HmL is the firms having a high B|M ratio (value) minus firms having a low B|M ratio (growth) called Value-factor. β_s and β_s are the coefficients of size and value factors.

6. Liquidity augmented FF-3FM (henceforth LFF-3FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_l(ImL) + \varepsilon_i \qquad (7)$$

7. Carhart (1997) four-factor model C-4FM) or Momentum augmented FF-3FM

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_w(WmL) + \varepsilon_i \qquad (8)$$

8. Liquidity & Momentum augmented FF-3FM (henceforth LMFF-3FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_l(ImL) + \beta_w(WmL) + \varepsilon_i \quad (9)$$

9. Fama & French (2015) five-factor model (henceforth FF-5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \varepsilon_i \quad (10)$$

In which, RMW is the firms having Robust profitability minus firms having Weak profitability returns called Profitability-factor. CMA is the firms having greater assets Conservative minus firms having lower assets Aggressive firms returns called Investment factor. β_p and β_i are the coefficients of profitability and investment factors respectively.

10. Liquidity augmented FF-5FM (henceforth LFF-5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \varepsilon_i \quad (11)$$

11. Momentum augmented FF-5FM (henceforth MFF-5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_w(WmL) + \varepsilon_i \quad (12)$$

12. Liquidity & Momentum augmented FF-5FM (henceforth LMFF-5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \beta_w(WmL) + \varepsilon_i \quad (13)$$

DATA ANALYSIS

This study forecasts the impact of various factors (liquidity and momentum adjusted FF5FM) on average portfolio stock returns (APSR) through time-series OLS regression techniques by following (Fama & French, 1993, 2015), to investigate the emerging market of Pakistan. For analyzing the PSX, this study used secondary data for the period Jan-2002 to Jun-2020. The empirical analyses are carried out using single and multivariate time series analysis to compare the liquidity and momentum augmented APMs to evaluate the accuracy of the APMs using PSX data.

Table 1 Descriptive Statistics and Covariance Matrix:

Panel-A: Descriptive Statistics:							
Variable	**RmRf**	**SMB**	**HML**	**WML**	**RMW**	**CMA**	**IML**
Mean	0.00772	-0.00861	-0.02418	0.05877	-0.00444	0.01838	0.00781
Std. Dev.	0.07465	0.04188	0.0415	0.05921	0.03296	0.04197	0.05459
Min	-0.45966	-0.1526	-0.27753	-0.14013	-0.09243	-0.1359	-0.18123
Max	0.23542	0.12892	0.1035	0.42918	0.16085	0.22976	0.12733
Obs.	228	228	228	228	228	228	228
Panel B: Covariance Matrix:							
RmRf	1						
SMB	-0.15994	1					
HML	0.30693	-0.12384	1				
WML	-0.22556	0.27243	-0.44339	1			
RMW	-0.26991	0.58738	-0.0947	0.26118	1		
CMA	-0.03058	-0.61254	-0.29642	-0.02929	-0.3859	1	
IML	0.3916	0.02226	0.547	-0.22153	-0.08628	-0.22923	1

Table 1, panel-A and B depict the descriptive statistics and covariance matrix for factors (market, size, value, momentum, profitability, investment, and liquidity) to identify the nexus magnitude and the strength between independent variables of the study. As the results demonstrate in Panel-A, the average monthly market premium (RmRf) determines 0.0077% for the time-span of 2002-2020 with a standard deviation (SD) of 0.0747 which ranges from -0.45966 to 0.23542. The size pattern (SMB) exhibits average returns of -0.0086 with an SD of 0.04188 having a range from -0.1526 to 0.12892. Similarly, HML, WML, RMW, CMA and IML show average monthly returns of -0.02418, 0.05877, -0.00444, 0.01838 and 0.00781 with SD 0.0415, 0.05921, 0.03296, 0.04197 and 0.05459 respectively. The momentum displays the highest average monthly returns of 0.05877 with the highest SD of 0.05921 and ranges from -0.14013 to 0.42918 which is the highest in comparison with other factors. Based on the mean returns of Fama-French five factors, market and size show positive and negative monthly returns respectively which are similar with (Zada et al., 2018) while the other three (value, profitability, and investment) show dissimilar results in terms of magnitude. The market, investment, and momentum show positive mean returns which are similar to (Ali et al., 2021).

Table 1, Panel-B indicates the correlation matrix and presents no strong multicollinearity between the independent variables. Panel-B shows that there is a positive association of RmRf with HML and IML, and negative with SMB, WML, RMW, and CMA. Similarly, HML shows an inverse correlation with WML, RMW, CMA while positive with IML. The correlation of SMB with HML and CMA displays negative while with WML, RMW, IML is positive. The WML shows weak negative nexus with CMA and IML while positive with RMW. Similarly, profitability shows inverse nexus with CMA and IML. Likewise, investment shows an inverse correlation with IML. Unpredictably, the CMA shows a negative association with all other factors such as RmRf, SMB, HML, WML, RMW, and IML.

Average Annual Returns based on Factors:

Based on the sample period from January 2002 through December 2020, table 2 summarizes the average annual returns for Market, Size, Value, Profitability, Investment, Momentum, and Liquidity factors. The last row reports the average returns for each factor based on the overall sample period, as follows:

Table 2: Average Annual Factor Returns (2002-2020):

Year	RmRf	SMB	HML	RMW	CMA	WML	IML
2002	5.769%	0.293%	-1.476%	-0.319%	0.193%	7.180%	0.696%
2003	4.043%	-1.758%	-0.510%	-1.867%	1.700%	6.324%	3.419%
2004	2.541%	1.558%	-1.058%	1.150%	-0.774%	7.078%	3.562%
2005	3.151%	-2.041%	-2.466%	-1.985%	3.278%	6.760%	1.343%
2006	-0.233%	0.039%	-3.653%	0.355%	0.903%	7.236%	-0.686%
2007	2.324%	-1.044%	-2.079%	0.035%	2.417%	7.791%	1.969%
2008	-8.085%	1.642%	-6.984%	2.648%	3.221%	12.420%	-6.101%
2009	3.092%	-5.964%	-3.157%	-4.034%	5.270%	-1.140%	-1.252%
2010	1.088%	-0.928%	-1.575%	0.400%	0.578%	-0.507%	0.659%
2011	-1.464%	-1.386%	-0.454%	-1.970%	1.205%	6.893%	4.195%

2012	2.479%	0.968%	-1.291%	0.709%	-0.007%	7.221%	3.274%
2013	2.604%	-2.085%	-2.594%	-1.890%	3.417%	6.472%	1.418%
2014	1.216%	-0.259%	-3.733%	0.318%	1.063%	7.235%	-1.006%
2015	-0.404%	0.175%	-2.544%	0.925%	2.275%	7.827%	1.677%
2016	2.668%	1.260%	-7.131%	1.306%	2.451%	12.243%	-7.098%
2017	-1.873%	-6.324%	-2.279%	-3.251%	5.431%	-1.627%	-0.010%
2018	-1.326%	-0.760%	-1.553%	0.419%	0.866%	-0.255%	0.600%
2019	-0.187%	-1.152%	-0.462%	-1.941%	0.824%	7.274%	4.658%
2020	-2.734%	1.412%	-0.949%	0.546%	0.619%	5.235%	3.513%
2002-2020	**0.772%**	**-0.861%**	**-2.418%**	**-0.444%**	**1.838%**	**5.877%**	**0.781%**

On average for the sample period 2002-2020, the momentum pattern shows the highest 5.877% average returns while the value pattern shows the lowest -2.418% average returns. Interestingly, the momentum displays the highest average annual returns (12.42%) for 2008 whereas the lowest (-1.63%) for 2017. The value pattern signposts overall negative annual average returns for the whole sample period. The CMA expresses positive average annual returns except 2004 and 2012. The liquidity pattern shows mixed results as the maximum average annual returns (4.66%) for the year 2019 while the minimum average annual returns (-7.10%) for the year 2016 which indicates the era before and after the general election. The market factor represents mix annual returns while the maximum (5.77%) for 2002 and the minimum (-8.09%) for 2008.

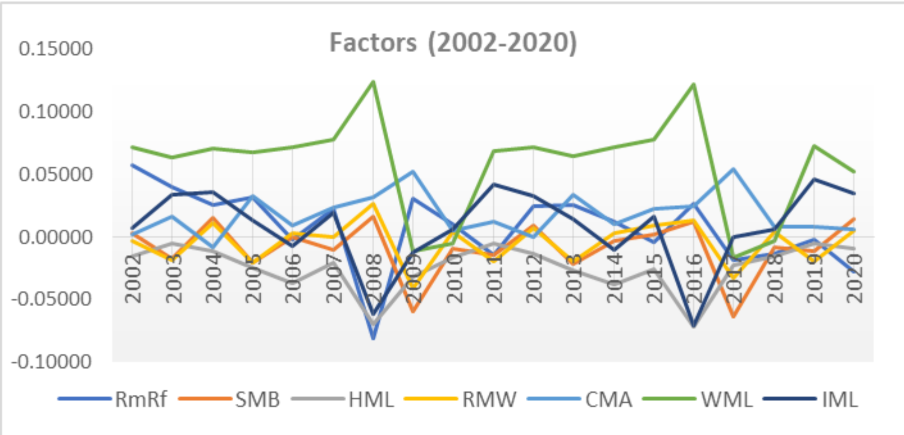

Figure 1: Combine graphical presentation of Factors (RmRf, SMB, HML, RMW, CMA, WML, and IML).

Figure 1 portrays the graphical presentation for all seven factors simultaneously used in this study. Based on similarity, all the factors represent the impact of the 2008 financial crisis as high volatility except the CMA factor. The liquidity pattern shows consistent volatility, which indicates the significant contribution, played in the emerging market. However, the momentum pattern shows more variations during the financial crises era.

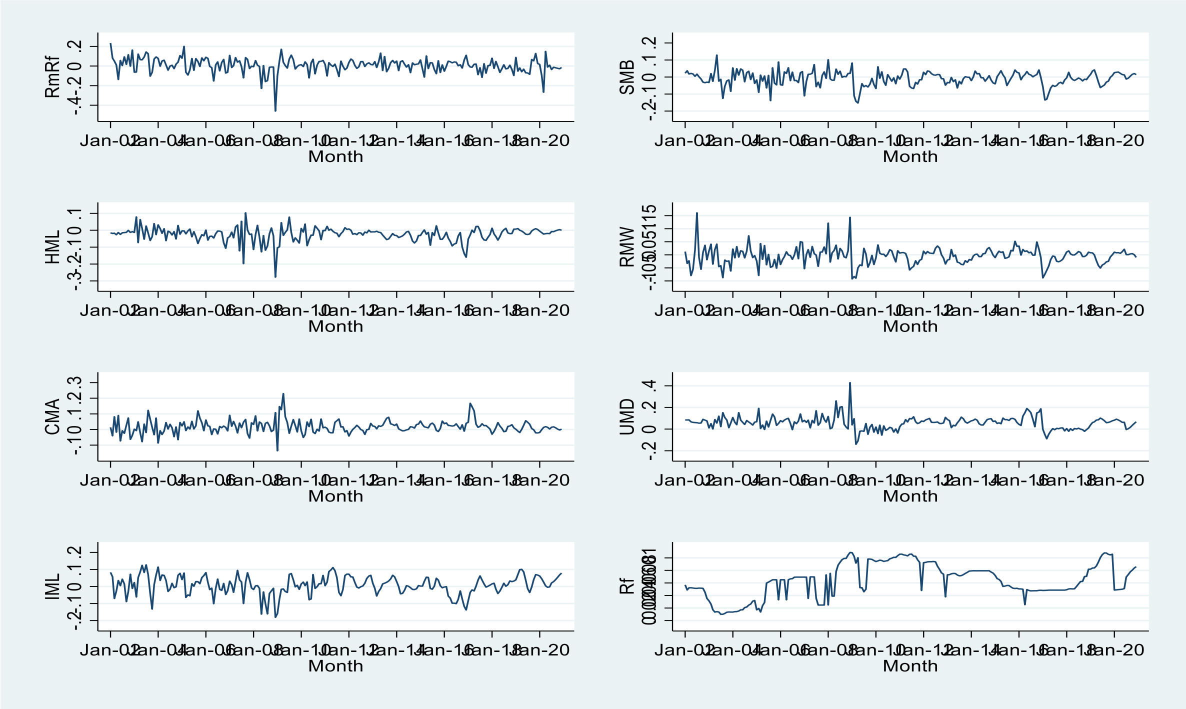

Figure 2: All Factors and Rf graphical presentation over 2002-2020.

Figure 2 demonstrates the patterns of all seven factors and risk-free rates used in this study. Based on similarity, all the factors depict the impact of the 2008 financial crisis as high volatility. The liquidity pattern shows consistent volatility, which indicates the significant contribution, played in the emerging market. Conversely, the momentum pattern shows less fluctuation except 2008 and 2017-the general election era.

Table 3: Factor Spanning Test Regression results

	(1) RmRf	(2) SMB	(3) HML	(4) WML	(5) IML	(6) RMW	(7) CMA
SMB	-0.122		0.058**	0.055	0.143***	-0.018	-0.004
	(-0.814)		(1.988)	(1.588)	(3.184)	(-0.678)	(-0.160)
HML	-0.040	0.205**		-0.303***	-0.066	-0.254***	0.309***
	(-0.142)	(1.988)		(-4.757)	(-0.777)	(-5.194)	(7.379)
WML	-0.064	0.137	-0.212***		0.789***	-0.045	0.223***
	(-0.269)	(1.588)	(-4.757)		(13.871)	(-1.050)	(6.238)
IML	-0.203	0.209***	-0.028	0.468***		0.030	-0.043
	(-1.117)	(3.184)	(-0.777)	(13.871)		(0.916)	(-1.487)
RMW	0.105	-0.076	-0.299***	-0.075	0.085		0.177***
	(0.342)	(-0.678)	(-5.194)	(-1.050)	(0.916)		(3.683)
CMA	0.099	-0.020	0.460***	0.475***	-0.155	0.224***	
	(0.286)	(-0.160)	(7.379)	(6.238)	(-1.487)	(3.683)	
RmRf		-0.016	-0.002	-0.003	-0.019	0.003	0.003
		(-0.814)	(-0.142)	(-0.269)	(-1.117)	(0.342)	(0.286)
_cons	0.004	-0.005**	0.001	0.008***	-0.005***	0.002*	-0.001
	(0.743)	(-2.406)	(0.990)	(7.521)	(-2.963)	(1.752)	(-0.960)
Obs.	336	336	336	336	336	336	336
R-squared	0.014	0.101	0.251	0.517	0.470	0.099	0.219
Adj. R2	-0.004	0.084	0.237	0.508	0.460	0.083	0.205
F	0.793	6.130	18.336	58.681	48.578	6.058	15.404

T-values are in parenthesis
**** p<0.01, ** p<0.05, * p<0.1*

We regress each of the seven factors on the other six factors intending to access which factor has unique information about expected returns and which factor is redundant, (Fama & French, 2017). Fama and French (2015) employed factor-spanning tests to show that if the intercept in a regression of one factor on other factors is close to zero, that factor is redundant. This study employs six factors in regressions to explain the seventh component. As the table-3 illustrates the redundancy or spanning test to compare the risk factors and to investigate whether individual factor contributes to the asset-pricing model. Market regression's alpha is 0.004, with a t-value of 0.743 and an R-square of 1%, indicating that the market is not described by other patterns. Similarly, the results of the factor-spanning test verified that size, liquidity, momentum, and profitability contributors since these patterns regressions yielded statistically significant intercepts. Therefore, these factors cannot be replicated by other patterns. On the contrary, market, value, and investment patterns regressions exhibit insignificant alpha values which proved poor contributions in the market. The value factor results show consistency with (Ali et al., 2021; Fama & French, 2015).

Table 4: SINGLE-FACTOR MODEL (CAPM):

DV	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2
SH	RmRf	0.4950	7.78	0.000	60.46	0.000	0.2076
	_cons	0.0007	0.14	0.888			
SM	RmRf	0.2877	5.76	0.000	33.14	0.000	0.124
	_cons	0.0152	4.06	0.000			
SL	RmRf	0.2824	5.16	0.000	26.63	0.000	0.1014
	_cons	-0.0042	-1.03	0.306			
BH	RmRf	0.2903	6.42	0.000	41.25	0.000	0.1506
	_cons	0.0154	4.55	0.000			
BM	RmRf	0.3501	6.25	0.000	39.03	0.000	0.1435
	_cons	-0.0050	-1.2	0.232			
BL	RmRf	0.2341	4.24	0.000	17.99	0.000	0.0697
	_cons	0.0126	3.04	0.003			

Note: The first column shows dependent variables (portfolio excess returns) 6 equal value-weighted portfolios constructed based on size and B|M ratio. The second column shows independent variables (market factor). The third, fourth, and fifth columns represent coefficient, t-statistic and corresponding p-value. The sixth, seventh and eighth columns show the overall OLS regression results, F-value, corresponding p-value, and adjusted R-square.

Table 4 demonstrates the single-factor (market-factor known as CAPM) using six equal-weighted portfolio returns. The results indicate statistically significant results for all portfolios. The F-values of all six models vary from 17.99 to 60.46, with a p-value of 0.000, indicating that the overall results are very significant. Similarly, the adjusted R-square ranges from 06.97% to 20.76%. The market coefficients also indicate statistically significant values with $p < 0.001$, which can be confirmed from corresponding t-statistics as it ranges from 4.24 to 7.78. Table 5, panel A presents the liquidity augmented CAPM (LCAPM) which indicates that after augmenting liquidity with market factor, the market factor results become statistically insignificant except portfolio SH ($\beta = 0.2003$, t-value = 4.18). The liquidity factor shows statistically highly significant results for all six portfolios with $p < 0.001$. The f-value also shows very high values while the adjusted R-square also shows growth in explaining average stock returns.

Table 5: TWO-FACTOR MODELS: (Liquidity augmented CAPM and Momentum augmented CAPM):

	Panel A: Liquidity augmented CAPM							Panel B: Momentum augmented CAPM						
DV	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2
SH	RmRf	0.2003	4.18	0.000	186.6	0.000	0.6206	RmRf	0.4303	6.87	0.000	43.36	0.000	0.2718
	IML	1.0290	15.7	0.000				WML	-0.3614	-4.6	0.000			
	_cons	-0.0051	-1.53	0.127				_cons	0.0224	3.4	0.001			
SM	RmRf	0.0173	0.6	0.552	340.3	0.000	0.7493	RmRf	0.2741	5.35	0.000	17.28	0.000	0.1254
	IML	0.9442	23.8	0.000				WML	-0.0756	-1.2	0.243			
	_cons	0.0099	4.93	0.000				_cons	0.0198	3.66	0.000			
SL	RmRf	-0.0380	-1.58	0.116	659.6	0.000	0.853	RmRf	0.2694	4.8	0.000	13.85	0.000	0.1017
	IML	1.1188	34	0.000				WML	-0.0729	-1	0.304			
	_cons	-0.0105	-6.27	0.000				_cons	0.0002	0.03	0.975			
BH	RmRf	0.0380	1.58	0.116	444.7	0.000	0.7963	RmRf	0.2571	5.66	0.000	26.75	0.000	0.1849
	IML	0.8812	26.8	0.000				WML	-0.1859	-3.2	0.001			
	_cons	0.0105	6.27	0.000				_cons	0.0266	5.56	0.000			
BM	RmRf	0.0692	1.82	0.069	229	0.000	0.6677	RmRf	0.2969	5.35	0.000	30.04	0.000	0.2037
	IML	0.9807	18.9	0.000				WML	-0.2974	-4.3	0.000			
	_cons	-0.0105	-4	0.000				_cons	0.0129	2.2	0.028			
BL	RmRf	-0.0350	-0.9	0.370	177.2	0.000	0.6082	RmRf	0.2287	4.03	0.000	9.05	0.000	0.0663
	IML	0.9397	17.7	0.000				WML	-0.0303	-0.4	0.672			
	_cons	0.0073	2.71	0.007				_cons	0.0144	2.41	0.017			

Panel B shows momentum augmented CAPM which indicates that the market factor show statistically significant coefficients for all portfolios as CAPM with $p < 0.001$. The momentum factor shows mixed results as portfolio SH, BH, and BM display statistically significant but inverse relationship with excess portfolio stocks returns (EPSR) while SM, SL, and BL show statistically inverse but insignificant nexus with EPSR. The F-values show significant values but are not as influential as LCAPM and same as adjusted R-square values. Conclusively, liquidity pattern demonstrates a highly significant impact on EPSR using equal-weighted portfolios returns in PSX but adversely influences the market pattern.

Table 6 panel A demonstrates the momentum and liquidity adjusted CAPM (alternative three-factor model – ATFM) and panel B shows FF3FM. The market factor indicates same insignificant mechanism as LCAPM whereas momentum shows statistically significant results except SM ($\beta = 0.0476$, t-value = 1.36, p-value = 0.174) using momentum and liquidity adjusted CAPM. Conversely, the FF3FM exhibits statistically significant coefficients for market and value patterns while the size pattern shows better results. The SM portfolio appearances inverse insignificant coefficient for size ($\beta = -0.0104$, t-value = -0.12, p-value = 0.904) whereas other portfolios show statistically significant coefficients which supports size-theory proposes small market-cap firms outperform big market-cap firms. The adjusted R-square and F-statistic values of ATFM demonstrate better results as compare to FF3FM in PSX. Similar to LCAPM, the market factor coefficients are inversely influenced after including liquidity into ATFM.

Table 7 panel A and B represent four-factor models named liquidity adjusted FF3FM (LFF3FM) and momentum adjusted FF3FM (also known as (Carhart, 1997) four-factor model - C4FM) respectively. The results of LFF3FM demonstrate statistically significant models' overall results as F-values for all six portfolios presents higher values with p-value < 0.001. At the micro-level, the market factor shows statistically insignificant coefficients except for portfolio SH ($\beta = 0.0445$, t-stat. = 3.07, p-value <0.005, similar to LCAPM and MLCAPM.

Table 6: THREE-FACTOR MODELS: (Liquidity and Momentum augmented CAPM and FF-3FM):

	Momentum & Liquidity augmented CAPM:							Fama & French (1992) three-factor model:						
DV	**IV**	**Coef.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**	**IV**	**Coef.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**
	RmRf	0.1700	3.64	0.000	139.5	0.000	0.6466	RmRf	0.3215	5.56	0.000	55.1	0.000	0.4169
SH	WML	-0.2334	-4.20	0.000				SMB	-0.2658	-2.69	0.008			
	IML	0.9891	15.5	0.000				HML	0.8771	8.47	0.000			
	_cons	0.0092	1.96	0.051				_cons	0.0209	4.22	0.000			
	RmRf	0.0235	0.80	0.425	228.3	0.000	0.7503	RmRf	0.2104	4.17	0.000	20.43	0.000	0.2044
SM	WML	0.0476	1.36	0.174				SMB	-0.0104	-0.12	0.904			
	IML	0.9523	23.8	0.000				HML	0.4475	4.96	0.000			
	_cons	0.0070	2.39	0.018				_cons	0.0265	6.13	0.000			
	RmRf	-0.0284	-1.18	0.239	452.9	0.000	0.8566	RmRf	0.1985	4.5	0.000	70.94	0.000	0.4803
SL	WML	0.0735	2.57	0.011				SMB	0.6331	8.4	0.000			
	IML	1.1313	34.4	0.000				HML	0.8245	10.5	0.000			
	_cons	-0.0150	-6.22	0.000				_cons	0.0218	5.77	0.000			
	RmRf	0.0284	1.18	0.239	306.1	0.000	8013	RmRf	0.1871	4.34	0.000	35.15	0.000	0.3109
BH	WML	-0.0735	-2.57	0.011				SMB	-0.3097	-4.2	0.000			
	IML	0.8687	26.4	0.000				HML	0.4420	5.73	0.000			
	_cons	0.0150	6.22	0.000				_cons	0.0242	6.54	0.000			
	RmRf	0.0465	1.25	0.212	167.7	0.000	0.6878	RmRf	0.1923	4.17	0.000	71.77	0.000	0.4833
BM	WML	-0.1744	-3.94	0.000				SMB	0.1600	2.03	0.044			
	IML	0.9510	18.7	0.000				HML	1.0089	12.2	0.000			
	_cons	0.0001	0.04	0.969				_cons	0.0220	5.55	0.000			
	RmRf	-0.0228	-0.58	0.560	121.1	0.000	0.6134	RmRf	0.1789	3.21	0.002	14.96	0.000	0.1557
BL	WML	0.0933	2.00	0.046				SMB	0.2399	2.52	0.012			
	IML	0.9556	17.9	0.000				HML	0.4496	4.51	0.000			
	_cons	0.0016	0.41	0.679				_cons	0.0259	5.43	0.000			

Table 7: FOUR-FACTOR MODELS (Liquidity and Momentum augmented FF3FM & Carhart 4FM):

	Panel A: Liquidity augmented FF-three-factor model							Panel B: Carhart (1997) four-factor model							
DV	**IV**	**Std. Err.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**	**DV**	**IV**	**Coef.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**
	RmRf	0.0445	3.07	0.002					RmRf	0.3164	5.46	0.000			
	SMB	0.0735	-5.72	0.000					SMB	-0.2384	-2.34	0.020			
SH	HML	0.0877	3.09	0.002	55.1	0.000	0.4169	SH	HML	0.8256	7.32	0.000	41.71	0.000	0.4177
	IML	0.0691	13.86	0.000					WML	-0.0913	-1.15	0.252			
	_cons	0.0040	-0.28	0.780					_cons	0.0253	4.05	0.000			
	RmRf	0.0283	0.36	0.720					RmRf	0.2139	4.22	0.000			
	SMB	0.0466	-3.82	0.000					SMB	-0.0293	-0.33	0.741			
SM	HML	0.0557	-3.78	0.000	20.43	0.000	0.2044	SM	HML	0.4830	4.91	0.000	15.52	0.000	0.2037
	IML	0.0438	23.69	0.000					WML	0.0630	0.91	0.364			
	_cons	0.0025	1.04	0.299					_cons	0.0235	4.3	0.000			
	RmRf	0.0153	0.22	0.829					RmRf	0.2016	4.56	0.000			
	SMB	0.0253	18.58	0.000					SMB	0.6166	7.95	0.000			
SL	HML	0.0302	6.08	0.000	70.94	0.000	0.4803	SL	HML	0.8555	9.94	0.000	53.37	0.000	0.4799
	IML	0.0237	42.64	0.000					WML	0.0548	0.91	0.366			
	_cons	0.0014	-1.08	0.282					_cons	0.0192	4.02	0.000			
	RmRf	0.0153	-0.22	0.829					RmRf	0.1865	4.3	0.000			
	SMB	0.0253	-18.6	0.000					SMB	-0.3062	-4.03	0.000			
BH	HML	0.0302	-6.08	0.000	35.15	0.000	0.3109	BH	HML	0.4355	5.17	0.000	26.26	0.000	0.308
	IML	0.0237	41.6	0.000					WML	-0.0116	-0.2	0.845			
	_cons	0.0014	1.08	0.282					_cons	0.0248	5.29	0.000			
	RmRf	0.0348	1.2	0.232					RmRf	0.1890	4.08	0.000			
	SMB	0.0574	0.59	0.557					SMB	0.1778	2.19	0.029			
BM	HML	0.0685	7.51	0.000	71.77	0.000	0.4833	BM	HML	0.9755	10.9	0.000	54.02	0.000	0.483
	IML	0.0539	14.49	0.000					WML	-0.0592	-0.93	0.351			
	_cons	0.0031	1.28	0.200					_cons	0.0248	4.97	0.000			
	RmRf	0.0392	-0.38	0.704					RmRf	0.1826	3.27	0.001			
	SMB	0.0647	1.19	0.233					SMB	0.2203	2.25	0.026			
BL	HML	0.0773	-2.42	0.016	14.96	0.000	0.1557	BL	HML	0.4864	4.48	0.000	11.39	0.000	0.1547
	IML	0.0608	16.53	0.000					WML	0.0653	0.85	0.395			
	_cons	0.0035	0.8	0.426					_cons	0.0228	3.78	0.000			

Conversely, the size pattern follows the theory and shows highly significant but inverse results for all small market-cap firms while insignificant for big market-cap firms (BM and BL). Additionally, the value pattern presents a statistically significant relationship with ASER. Correspondingly, the liquidity pattern shows a statistically significant nexus with ASER in the market. The presence of liquidity confirms an inverse impact on market factor as LCAPM and MLCAPM.

Panel B shows C-4FM regression results. Similar to MCAPM, the F-statistics and associated p-values of all models show statistically significant results. The market results show a high significance for all portfolios with a p-value < 0.005. Similarly, size and value patterns demonstrate significant nexus with portfolio returns. In contrast, momentum pattern shows insignificant for all portfolios and mix results as portfolios SH, BH and BM show negative nexus with ASPR, similar with (Khan, et al., 2021).

Table 8: FIVE-FACTOR MODELS: (Alternative five-factor model & Fama-French five-factor model):

Panel A: Momentum & Liquidity FF-3FM (Alternative 5FM)								Panel B: Fama & French (2015) 5FM						
DV	**IV**	**Coef.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**	**IV**	**Coef.**	**t**	**P>|t|**	**F-Val.**	**P>|F|**	**Adj. R-2**
	RmRf	0.130	2.93	0.0040	101.7	0.000	0.6892	RmRf	0.1813	4.42	0.000	119	0.000	0.7221
	SMB	-0.388	-5.16	0.0000				SMB	0.5672	5.5	0.000			
SH	HML	0.205	2.2	0.0290				HML	0.9714	12.12	0.000			
	WML	-0.112	-1.93	0.0550				RMW	-1.6879	-15.6	0.000			
	IML	0.961	13.99	0.0000				CMA	0.1322	1.37	0.173			
	_cons	0.004	0.87	0.3840				_cons	0.0215	6.25	0.000			
	RmRf	0.013	0.45	0.6560	155.6	0.000	0.773	RmRf	0.1878	3.78	0.000	17.51	0.000	0.2667
	SMB	-0.190	-3.97	0.0000				SMB	0.4197	3.35	0.001			
SM	HML	-0.186	-3.12	0.0020				HML	0.6166	6.34	0.000			
	WML	0.041	1.1	0.2740				RMW	-0.3327	-2.53	0.012			
	IML	1.037	23.67	0.0000				CMA	0.4268	3.64	0.000			
	_cons	0.001	0.23	0.8200				_cons	0.0252	6.02	0.000			
	RmRf	0.005	0.35	0.7270	757.8	0.000	0.9434	RmRf	0.1746	4	0.000	49.47	0.000	0.5164
	SMB	0.460	17.78	0.0000				SMB	0.4584	4.17	0.000			
SL	HML	0.203	6.29	0.0000				HML	0.6759	7.92	0.000			
	WML	0.033	1.65	0.1000				RMW	-0.2224	-1.92	0.056			
	IML	1.011	42.75	0.0000				CMA	-0.4112	-3.99	0.000			
	_cons	-0.003	-1.83	0.0680				_cons	0.0235	6.4	0.000			
	RmRf	-0.005	-0.35	0.7270	534.6	0.000	0.9216	RmRf	0.1915	4.44	0.000	25.12	0.000	0.347
	SMB	-0.460	-17.8	0.0000				SMB	-0.0527	-0.49	0.627			
BH	HML	-0.203	-6.29	0.0000				HML	0.5863	6.96	0.000			
	WML	-0.033	-1.65	0.1000				RMW	-0.0054	-0.05	0.963			
	IML	0.989	41.79	0.0000				CMA	0.3837	3.78	0.000			
	_cons	0.003	1.83	0.0680				_cons	0.0228	6.29	0.000			
	RmRf	0.037	1.07	0.2880	126.8	0.000	0.7348	RmRf	0.1879	4.35	0.000	61.26	0.000	0.5703
	SMB	0.056	0.96	0.3390				SMB	0.6761	6.22	0.000			
BM	HML	0.470	6.42	0.0000				HML	1.2668	15.02	0.000			
	WML	-0.076	-1.68	0.0950				RMW	-0.1533	-1.34	0.181			
	IML	0.783	14.58	0.0000				CMA	0.6757	6.64	0.000			
	_cons	0.008	2.02	0.0450				_cons	0.0196	5.4	0.000			
	RmRf	-0.012	-0.31	0.7560	74.57	0.000	0.6184	RmRf	0.1647	3.33	0.001	27.58	0.000	0.3693
	SMB	0.064	0.97	0.3330				SMB	1.0205	8.21	0.000			
BL	HML	-0.162	-1.95	0.0520				HML	0.8213	8.5	0.000			
	WML	0.044	0.85	0.3980				RMW	-0.3145	-2.4	0.017			
	IML	1.004	16.49	0.0000				CMA	0.9671	8.3	0.000			
	_cons	0.001	0.17	0.8650				_cons	0.0226	5.44	0.000			

Table 8 consists of panel A which presents the momentum and liquidity adjusted FF3FM (ML-FF3FM) and panel B which shows (FF5FM). The MLFF5FM shows a statistically highly significant nexus with ASPR as F-value and its associated p-values indicate better results. The market factor results show clear insignificant results consistent with LCAPM, MLCAPM, and LFF3FM results. The findings suggest that liquidity inversely influences the coefficients of market pattern consistently. The size factor indicates significant coefficients for small market-cap stocks portfolios while two big market-cap stocks portfolios (BM and BL), similar to (Ali et al., 2021; Khan & Iqbal, 2021). The value-factor indicates statistically significant coefficients except portfolio BL (β = -0.162, t-value = -1.95 with p-value = 0.0520). The momentum pattern shows overall insignificant results in the market, consistent with (Ali et al., 2021). Conversely, liquidity demonstrates consistently statistically significant results for the portfolios in the market. The FF5FM indicates statistically significant results in the market as F-values and their corresponding p-values show highly statistically significant findings. In-depth, market, and value factors demonstrate highly significant coefficients for all portfolios. The size-factor also displays significant coefficients except portfolio BH (β = -0.0527, t-value = -0.49 and p-value = 0.627). Similarly, the profitability pattern shows mixed results as BH and BM show statistically insignificant but negative nexus with ASPR. The investment-pattern shows positive and statistically significant results except portfolio BH (β = 0.1322, t-value = 1.37 and p-value = 0.173 which is inconsistent with (Khan & Iqbal, 2021). The findings conclude that both five-factor models MLFF3FM and FF5FM produce influential and significant results in PSX using six equal-weighted portfolios. As compare to FF-3FM, FF-5FM outperformed which is consistent with (Khan & Iqbal, 2021).

Table 9: SIX-FACTOR MODELS: OLS Regression Results for Momentum & Liquidity augmented FF-5FM:

	Panel A: Momentum augmented FF-5FM							Panel B: Liquidity (Liu, 2006)augmented FF-5FM						
DV	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2
	RmRf	0.1817	4.42	0.000	98.72	0.000	0.7209	RmRf	0.0177	1.06	0.288	863.55	0.000	0.958
	SMB	0.5653	5.44	0.000				SMB	0.3860	9.54	0.000			
	HML	0.9781	11.29	0.000				HML	0.4074	11.63	0.000			
SH	RMW	-1.6905	-15.4	0.000				RMW	-1.5861	-37.5	0.000			
	CMA	0.1327	1.37	0.172				CMA	0.1445	3.84	0.000			
	WML	0.0115	0.21	0.836				IML	0.8936	35.32	0.000			
	_cons	0.0210	4.81	0.000				_cons	0.0009	0.58	0.562			
	RmRf	0.1912	3.85	0.000	14.96	0.000	0.2695	RmRf	-0.0009	-0.04	0.970	185.29	0.000	0.8297
	SMB	0.4039	3.22	0.001				SMB	0.2105	3.46	0.001			
	HML	0.6702	6.41	0.000				HML	-0.0341	-0.65	0.518			
SM	RMW	-0.3536	-2.67	0.008				RMW	-0.2153	-3.39	0.001			
	CMA	0.4305	3.68	0.000				CMA	0.4410	7.81	0.000			
	WML	0.0916	1.37	0.172				IML	1.0310	27.11	0.000			
	_cons	0.0208	3.94	0.000				_cons	0.0013	0.6	0.552			
	RmRf	0.1769	4.05	0.000	41.43	0.000	0.5166	RmRf	-0.0097	-0.96	0.340	1540.9	0.000	0.976
	SMB	0.4478	4.06	0.000				SMB	0.2542	10.31	0.000			
	HML	0.7120	7.74	0.000				HML	0.0405	1.9	0.059			
SL	RMW	-0.2364	-2.03	0.043				RMW	-0.1077	-4.18	0.000			
	CMA	-0.4087	-3.97	0.000				CMA	-0.3974	-17.3	0.000			
	WML	0.0617	1.05	0.296				IML	1.0068	65.24	0.000			
	_cons	0.0205	4.43	0.000				_cons	0.0002	0.21	0.835			
	RmRf	0.1913	4.42	0.000	20.84	0.000	0.344	RmRf	0.0097	0.96	0.340	1101.9	0.000	0.9668
	SMB	-0.0519	-0.48	0.635				SMB	-0.2542	-10.3	0.000			
	HML	0.5837	6.41	0.000				HML	-0.0405	-1.9	0.059			
BH	RMW	-0.0043	-0.04	0.970				RMW	0.1077	4.18	0.000			
	CMA	0.3835	3.76	0.000				CMA	0.3974	17.34	0.000			
	WML	-0.0046	-0.08	0.937				IML	0.9932	64.36	0.000			
	_cons	0.0230	5.02	0.000				_cons	-0.0002	-0.21	0.835			
	RmRf	0.1865	4.31	0.000	50.99	0.000	0.5692	RmRf	0.0450	1.55	0.122	174.24	0.000	0.8208
	SMB	0.6828	6.25	0.000				SMB	0.5178	7.32	0.000			
	HML	1.2442	13.66	0.000				HML	0.7743	12.65	0.000			
BM	RMW	-0.1445	-1.26	0.211				RMW	-0.0645	-0.87	0.384			
	CMA	0.6741	6.62	0.000				CMA	0.6864	10.44	0.000			
	WML	-0.0387	-0.66	0.507				IML	0.7805	17.64	0.000			
	_cons	0.0214	4.67	0.000				_cons	0.0015	0.59	0.554			
	RmRf	0.1685	3.41	0.001	23.52	0.000	0.3732	RmRf	-0.0184	-0.69	0.491	186.49	0.000	0.8306
	SMB	1.0028	8.05	0.000				SMB	0.8176	12.58	0.000			
	HML	0.8813	8.49	0.000				HML	0.1900	3.38	0.001			
BL	RMW	-0.3379	-2.57	0.011				RMW	-0.2006	-2.95	0.003			
	CMA	0.9712	8.36	0.000				CMA	0.9808	16.24	0.000			
	WML	0.1025	1.54	0.124				IML	1.0003	24.61	0.000			
	_cons	0.0177	3.37	0.001				_cons	-0.0006	-0.24	0.808			

Table 9 demonstrates the momentum and liquidity adjusted FF5FM (MFF5FM and LFF5FM respectively) by employing the time-series regression technique. Panel A illustrates the MFF5FM results as F-value and associated p-values show statistically significant values. The momentum factor demonstrates mix and insignificant results for all six portfolios similar to other models' results, similar with (Ali et al., 2021). The size-factor also displays significant coefficients except portfolio BH (β = -0.0519, t-value = -0.48 and p-value = 0.635) while the investment factor also shows significant coefficients results except portfolio SH (β = 0.1327, t-value = 1.37 and p-value = 0.172) almost similar to FF5FM findings. Similarly, the value pattern also shows a statistically significant relationship with APSR similar to FF5FM. Likewise, the profitability pattern shows mixed results as BH and BM show statistically insignificant but negative nexus with ASPR. The investment pattern shows positive and statistically significant results for all six portfolios in PSX. The FF-5FM outperformed as compare to momentum augmented FF-5FM which is similar to (Ali et al., 2021; Khan & Iqbal, 2021).

On the other hand, panel B presents the LFF-5FM regression results. Consistently, the inclusion of liquidity factor unfavorably influences the market factor returns, same in this model, the market pattern shows statistically insignificant results for the whole portfolios consistent with LCAPM, MLCAPM, LFF-3FM, and MLFF-3FM results. The size factor indicates statistically highly significant coefficients for all portfolios. The value factor shows mix as three portfolios show insignificant (SM, SL, and BH) and three significant results. The profitability-pattern shows significant but negative coefficients excluding portfolio BM (β = -0.0645, t-value = -0.87 and p-value = 0.384). Furthermore, the investment factor shows a positive and highly significant relationship with APSR in the market. Both the model's findings show similarity with the previously investigated models which support that market behavior for the factors are hypothesized correspondingly in the line of models as suggested by baseline models.

Table 10 demonstrates the liquidity and momentum adjusted FF5FM (LMFF-5FM - seven-factor model) in PSX using the time-series OLS regression technique. As the findings evidenced above that liquidity unfavorably divert the market factor returns from significant into insignificant, similar is the situation in LMFF5FM. The market factor shows statistically insignificant findings due to the existence of liquidity in the model. Conversely, the size factor shows a highly statistically significant nexus with APSR similar to the above findings. Similarly, the value-pattern depicts significant nexus with APSR except portfolio SM (β = .0039045, t-value = 0.07 and p-value = 0.944). Moreover, the profitability-factor shows statistically significant coefficients excluding BM portfolio (β = .0502531, t-value = -0.68 and p-value = 0.499).The coefficients of investment and liquidity-factor show a statistically significant impact on APSR in the market. The momentum pattern demonstrates mixed results as SH and BM show insignificant results while SM shows weak significant results. The F-values of all models show highly significant results with p-value = 0.000 and adjusted R-square ranges from 82.21% to 97.66%. Conclusively, the overall results validate the contribution of FF5FM while market and momentum factors exhibit statistically insignificant outputs in the market.

Model Diagnostics: GRS test:

The GRS test showed that the APMs under consideration appropriately forecast the average portfolio stocks returns. Based on the null hypothesis that the separate alphas are mutually equal to zero, the GRS test confirmed that FF5FM is an efficient model than CAPM, FF3FM, and C4FM for bitterly forecasting the APSR in PSX. APMs are very crucial and beneficial tools for investors and portfolio managers to forecast the stock returns in the equity market/s. It chooses the best explaining model based on average absolute alpha closed to zero, (Khan & Iqbal, 2021). Based on mean absolute alpha the LMFF5FM is the best model to explain the average portfolio stocks returns in the market. Similarly, the second appropriate model is LCAPM while the third model is LFF5FM based on GRS test findings.

Table 10: SEVEN-FACTOR MODEL: (Liquidity & Momentum augmented FF5FM):

DV	IV	Coef.	t	P>|t|	F-Val.	P>|F|	Adj. R-2
SH	RmRf	.0170535	1.02	0.307	738.32	0.000	0.9579
	SMB	.3882856	9.55	0.000			
	HML	.3988084	10.65	0.000			
	RMW	-1.582829	-37.12	0.000			
	CMA	.1439575	3.82	0.000			
	IML	.8941946	35.27	0.000			
	WML	-.0140408	-0.65	0.515			
	_cons	.001513	0.85	0.397			
SM	RmRf	.0018103	0.07	0.942	161.34	0.000	0.8318
	SMB	.2002869	3.30	0.001			
	HML	.0039045	0.07	0.944			
	RMW	-.229737	-3.61	0.000			
	CMA	.4434575	7.90	0.000			
	IML	1.028574	27.20	0.000			
	WML	.0622393	1.94	0.054			
	_cons	-.0016283	-0.61	0.541			
SL	RmRf	-.0082194	-0.82	0.413	1354.45	0.000	0.9766
	SMB	.2487723	10.17	0.000			
	HML	.0606205	2.69	0.008			
	RMW	-.1154053	-4.50	0.000			
	CMA	-.3960621	-17.49	0.000			
	IML	1.005483	65.92	0.000			
	WML	.0329753	2.55	0.012			
	_cons	-.0013714	-1.28	0.202			
BH	RmRf	.0082194	0.82	0.413	968.82	0.000	0.9676
	SMB	-.2487723	-10.17	0.000			
	HML	-.0606205	-2.69	0.008			
	RMW	.1154054	4.50	0.000			
	CMA	.3960621	17.49	0.000			
	IML	.9945169	65.20	0.000			
	WML	-.0329754	-2.55	0.012			
	_cons	.0013714	1.28	0.202			
BM	RmRf	.0423264	1.46	0.146	150.85	0.000	0.8221
	SMB	.527799	7.46	0.000			
	HML	.7369962	11.31	0.000			
	RMW	-.0502531	-0.68	0.499			
	CMA	.683977	10.44	0.000			
	IML	.7829061	17.75	0.000			
	WML	-.061057	-1.63	0.104			
	_cons	.0043973	1.42	0.158			
BL	RmRf	-.0151476	-0.57	0.568	163.17	0.000	0.8334
	SMB	.8053917	12.45	0.000			
	HML	.2351572	3.95	0.000			
	RMW	-.2177896	-3.21	0.002			
	CMA	.9837481	16.42	0.000			
	IML	.9973718	24.72	0.000			
	WML	.0740034	2.16	0.032			
	_cons	-.0040648	-1.43	0.153			

Table 11: GRS test results:

Model	GRS F-test	GRS p-value	Mean Alpha	Mean SE	Mean R2	Mean adj. R2
CAMP	15.793596	0.000000	0.005771	0.004053	0.136613	0.132793
LCAPM	15.083784	0.000000	0.000274	0.002333	0.718352	0.715848
MCAPM	8.511414	0.000000	0.016032	0.005748	0.166383	0.158973
LMCAPM	6.966511	0.000001	0.002992	0.003339	0.729611	0.725990
FF3FM	7.663603	0.000000	0.023572	0.004252	0.350619	0.341922
LFF3FM	0.600368	0.729898	0.001383	0.002640	0.782832	0.778936
MFF3FM	5.445402	0.000029	0.023403	0.005370	0.352783	0.341173
LMFF3FM	1.335329	0.242556	0.002205	0.003208	0.784901	0.780057
FF5FM	7.099849	0.000001	0.022529	0.003783	0.477057	0.465279
LFF5FM	0.158933	0.987073	0.000517	0.001725	0.899690	0.896967
MFF5FM	4.889960	0.000104	0.020732	0.004779	0.479702	0.465576
LMFF5FM	0.964079	0.450507	0.000036	0.002087	0.901352	0.898213

Table 11 shows the GRS test results for all models using equal-weighted six Size-B|M double-sorted portfolios using data of PSX for the period 2002-2020. The results include F-test, p-value, average-intercept, mean standard errors, average R-square, and average adjusted R-square.

5. Conclusion

This study aims to examine the seven-factor model, which consists of multidimensional liquidity and momentum augmented FF-5FM using monthly data from Jan-2002 to Dec-2020 to examine whether liquidity and momentum in a combination of FF-5FM contribute to the stock returns determination in the Pakistan Stock Exchange. To the best of our knowledge, Liu's (2006) multidimensional liquidity and momentum collectively are augmented with standard asset-pricing models CAPM, FF-3FM, C-4FM, and FF-5FM are used for the first time in one study in an emergent stock market like Pakistan. This study compared and evaluated the performance of predictability in the emerging equity market using 12 asset-pricing models. Moreover, the factor-spanning test in PSX verified that liquidity and momentum are important augmentations to FF-5FM since both pattern regressions yielded statistically significant intercepts. Similarly, the liquidity and momentum augmented FF-5FM over-performs, as the absolute average alpha (AAA = 0.000036) of the GRS test suggests that LM-FF5FM offered a superior explanation for Size-B|M portfolio returns. This verifies the spanning tests by demonstrating that Liquidity and Momentum have different beneficial features when augmented with the FF-5FM. Moreover, the second and third superior explanation models recommended by the GRS test are (Liu, 2006) two-factor model and Liquidity augmented FF-5FM respectively in PSX. Our results suggest that liquidity and momentum risks are important, and particularly the liquidity augmented APMs are preferred for investment decisions, financial market research, and regulation. More importantly, all the models significantly passed the GRS F-test.

Assuming that the FF-5FM performs well, this study concludes that the seven-factor model is a potential substitute to the FF-5FM. Overall, the results highlight the practical significance of liquidity and momentum risk factors in APMs. The main critic of FF-5FM is that the value factor is redundant while this study observed value-factor is not redundant. More particularly, the Liu multidimensional liquidity risk factor is an adequate factor that is practically relevant in investment portfolio management, portfolio construction, and financial economics which produces productive results of market efficiency.

For future recommendations, there are various factors such as leverage, P/E ratio, value-at-risk risk factors which may be augmented with FF-5FM to investigate further robust results in the market using a large period.

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